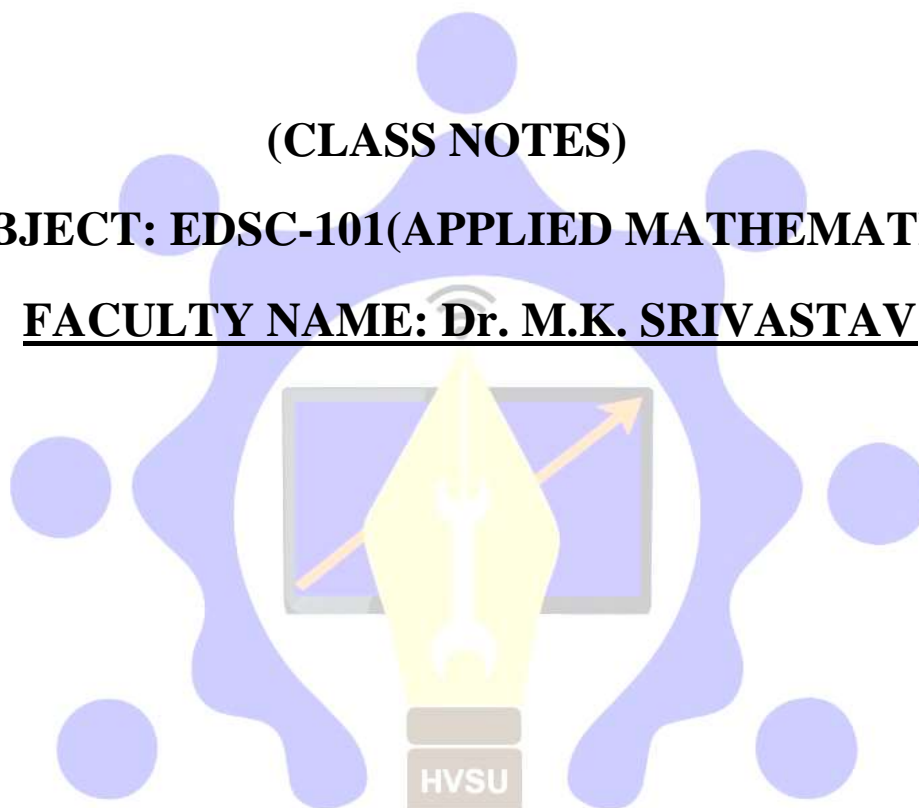


# **HARYANA VISHWAKARMA SKIL UNIVERSITY**

**(CLASS NOTES)**

**SUBJECT: EDSC-101(APPLIED MATHEMATICS)**

**FACULTY NAME: Dr. M.K. SRIVASTAV**



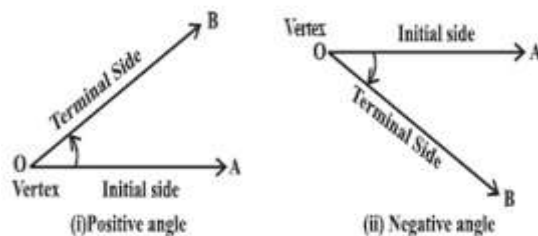
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## UNIT-2: TRIGONOMETRIC FUNCTIONS

**Angles:** Angle is a measure of rotation of a ray about its initial point. The original ray is called the *initial side* and the final position of the ray after rotation is called the *terminal side* of the angle. The point of rotation is called the *vertex*. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is *negative*.



**Degree Measure:** If a rotation from the initial side is  $\left(\frac{1}{360}\right)^{\text{th}}$  of a revolution, the angle is said to have a measure of one degree, written as  $1^\circ$ . A degree is divided into 60 minutes, and a minute is divided into 60 seconds.  $\left(\frac{1}{60}\right)^{\text{th}}$  of a degree is called a *minute*, written as  $1'$ , and one sixtieth of a minute is called a *second*, written as  $1''$ . Thus,  $1^\circ = 60'$ ,  $1' = 60''$ ,

- Radian measure  $= \frac{\pi}{180} \times \text{Degree measure}$
- Degree measure  $= \frac{180}{\pi} \times \text{Radian measure}$

**Radian Measure:** Another unit for measurement of an angle is called the *radian* measure.

$$2\pi \text{ radian} = 360^\circ \text{ or } \pi \text{ radian} = 180^\circ.$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ 16' \text{ approximately.}$$

$$\text{Also, } 1^\circ = \frac{\pi}{180} \text{ radian} = \frac{22}{7} \times \frac{1}{180} \text{ radian} = 0.01746 \text{ radian approximately.}$$

**Example:** The relation between degree measures and radian measure of some common angles are given in the following table:

Degree	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$

**Example:** Convert  $40^\circ 20'$  into radian measure.

**Solution:** We know that  $180^\circ = \pi$  radian

$$\text{Hence, } 40^\circ 20' = 40\frac{1}{3} \text{ degree} = \frac{\pi}{180} \times \frac{121}{3} \text{ radian} = \frac{121\pi}{540} \text{ radian}$$

**Example:** Convert 6 radians into degree measure.

**Solution:** we know that  $\pi$  radian =  $180^\circ$

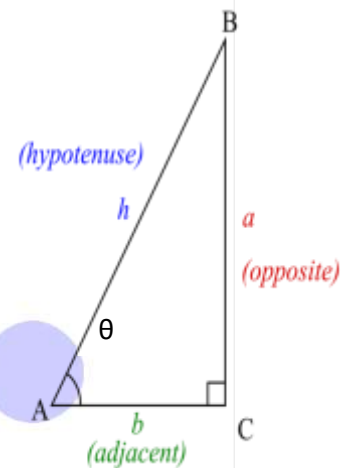
$$\begin{aligned} 6 \text{ radians} &= \frac{180}{\pi} \times 6 \text{ degree} = \frac{1080 \times 7}{22} \text{ degree} \\ &= 343 \frac{7}{11} \text{ degree} = 343^\circ + \frac{7 \times 60}{11} \text{ minute} \\ &= 343^\circ + 38' + \frac{2}{11} \text{ minute} \\ &= 343^\circ + 38' + 10.9'' \\ &= 343^\circ 38' 11'' \text{ approximately} \end{aligned}$$

### Topic 3.1 Trigonometry Function:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{hypotenuse}}{\text{opposite}}, \quad \sec \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{adjacent}}, \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\text{adjacent}}{\text{opposite}}$$

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \operatorname{cosec}^2 x$
- $\sin(-x) = -\sin x, \quad \cos(-x) = \cos x$
- $\tan(-x) = -\tan x, \quad \cot(-x) = -\cot x$
- $\sec(-x) = -\sec x, \quad \operatorname{cosec}(-x) = -\operatorname{cosec} x$



### Topic 3.2 Review of ratio of some standard angles:

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$
$\cot \theta$	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
$\operatorname{cosec} \theta$	$\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

- $\sin(\frac{\pi}{2} - x) = \cos x$ ,  $\cos(\frac{\pi}{2} - x) = \sin x$
- $\tan(\frac{\pi}{2} - x) = \cot x$ ,  $\cot(\frac{\pi}{2} - x) = \tan x$
- $\sec(\frac{\pi}{2} - x) = \operatorname{cosec} x$ ,  $\operatorname{cosec}(\frac{\pi}{2} - x) = \sec x$
- $\sin(2\pi - x) = -\sin x$ ,  $\cos(2\pi - x) = \cos x$
- $\tan(2\pi - x) = -\tan x$ ,  $\cot(2\pi - x) = -\cot x$
- $\sec(2\pi - x) = \sec x$ ,  $\operatorname{cosec}(2\pi - x) = -\operatorname{cosec} x$

The signs of other trigonometric functions in different quadrants are as follows:

	I	II	III	IV
$\sin x$	+	+	-	-
$\cos x$	+	-	-	+
$\tan x$	+	-	+	-
$\operatorname{cosec} x$	+	+	-	-
$\sec x$	+	-	-	+
$\cot x$	+	-	+	-

### **Topic 3.3 Addition, Subtraction and Product Formulae:**

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ ,  $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$ ,  $\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

**Example:** If  $\cos x = -\frac{3}{5}$ ,  $x$  lies in the third quadrant, find the values of other five trigonometric functions.

**Solution:** Since  $\cos x = -\frac{3}{5}$ , we have  $\sec x = \frac{1}{\cos x} = -\frac{5}{3}$

Now

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow \sin x = \pm \frac{4}{5}$$

Since  $x$  lies in the third quadrant,  $\sin x$  is negative. Therefore  $\sin x = -\frac{4}{5}$

Therefore  $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$

Further, we have  $\tan x = \frac{\sin x}{\cos x} = \frac{4}{3}$  and  $\cot x = \frac{1}{\tan x} = \frac{3}{4}$

**Example:** Find the value of  $\cos(-1710^\circ)$

**Solution:** We know that values of  $\cos x$  repeat after an interval of  $2\pi$  or  $360^\circ$ .

Therefore,  $\cos(-1710^\circ) = \cos(-1710^\circ + 5 \times 360^\circ) = \cos 90^\circ = 0$

**Example:** Prove that  $3\sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4\sin \frac{5\pi}{6} \cot \frac{\pi}{4} = 1$

**Solution:** We have,  $\text{LHS} = 3\sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4\sin \frac{5\pi}{6} \cot \frac{\pi}{4}$   
 $= 3 \times \frac{1}{2} \times 2 - 4\sin \left( \pi - \frac{\pi}{6} \right) \times 1 = 3 - 4\sin \frac{\pi}{6}$   
 $= 3 - 4 \times \frac{1}{2} = 1 = \text{RHS}$

### **Topics 3.4 Transformation from Product to Sum or Difference of Two Angles or Vice-Versa**

- $\sin x + \sin y = 2\sin \frac{x+y}{2} \cos \frac{x-y}{2}$
- $\sin x - \sin y = 2\sin \frac{x-y}{2} \cos \frac{x+y}{2}$
- $\cos x + \cos y = 2\cos \frac{x+y}{2} \cos \frac{x-y}{2}$
- $\cos x - \cos y = 2\sin \frac{x+y}{2} \sin \frac{y-x}{2}$
- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
- $2\sin x \sin y = \cos(x-y) - \cos(x+y)$

**Topics 3.5 Multiple and Submultiples Angles:**

- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$
- $\sin 2x = 2\sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$
- $\tan 2x = \frac{2 \tan x}{1 + \tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

**Topic 3.6 sin and cosine Formula, Area of Triangle**

**Sine rule:** In a right angled,  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

**Cosine rule:** In a right angled,  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of a Triangle:** The formula for the area of a triangle is

$$A_{\Delta} = \frac{1}{2}bh$$

By using the trigonometry in the given right angled triangle, we find:  $\sin C = \frac{h}{b} \Rightarrow h = b \sin C$  Therefore,

$$A_{\Delta} = \frac{1}{2}ab \sin C$$

**Example:** Show that  $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

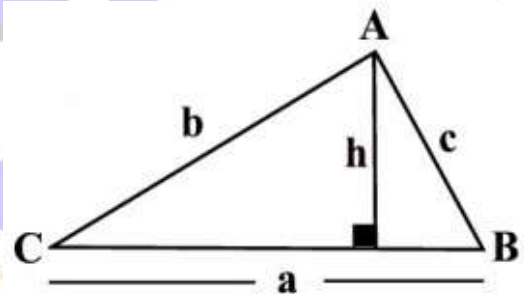
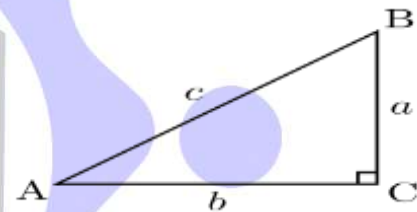
**Solution:**

$$\tan 3x = \tan(2x + x) \Rightarrow \tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$\tan 3x(1 - \tan 2x \tan x) = \tan 2x + \tan x$$

$$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$

$$\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$$



Example: Prove that  $\frac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \cot x$

Answer:

$$\begin{aligned} \text{LHS} &= \frac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \frac{2 \cos \left( \frac{7x+5x}{2} \right) \cos \left( \frac{7x-5x}{2} \right)}{2 \cos \left( \frac{7x+5x}{2} \right) \sin \left( \frac{7x-5x}{2} \right)} \\ &= \frac{\cos x}{\sin x} = \cot x = \text{RHS} \end{aligned}$$

Example: Prove that  $\cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}$

Solution: We have

$$\begin{aligned} \text{LHS} &= \cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} \\ &= \frac{2 \cos 2x \cos \frac{x}{2}}{2} - \frac{2 \cos \frac{9x}{2} \cos 3x}{2} \\ &= \frac{1}{2} \left[ \cos \left( 2x + \frac{x}{2} \right) + \cos \left( 2x - \frac{x}{2} \right) - \cos \left( \frac{9x}{2} + 3x \right) - \cos \left( \frac{9x}{2} - 3x \right) \right] \\ &= \frac{1}{2} \left[ \cos \frac{5x}{2} + \cos \frac{3x}{2} - \cos \frac{15x}{2} - \cos \frac{3x}{2} \right] \\ &= \frac{1}{2} \left[ \cos \frac{5x}{2} - \cos \frac{15x}{2} \right] = \frac{1}{2} \left[ -2 \sin \left\{ \frac{\frac{5x}{2} + \frac{15x}{2}}{2} \right\} \sin \left\{ \frac{\frac{5x}{2} - \frac{15x}{2}}{2} \right\} \right] \\ &= -\sin 5x \sin \left( -\frac{5x}{2} \right) = \sin 5x \sin \frac{5x}{2} = \text{RHS} \end{aligned}$$

Example: Find the value of  $\tan \frac{\pi}{8}$

Solution: Let  $x = \frac{\pi}{8}$ , Then  $2x = \frac{\pi}{4} \Rightarrow \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \Rightarrow \tan \frac{\pi}{4} = \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

$$\Rightarrow y = \tan \frac{\pi}{8}, \text{ then } 1 = \frac{2y}{1 - y^2} \Rightarrow y^2 + 2y - 1 = 0$$

$$\text{Therefore } y = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Since  $\frac{\pi}{8}$  lies in the first quadrant,  $y = \tan \frac{\pi}{8}$  is positive

$$\text{Hence, } \tan \frac{\pi}{8} = \sqrt{2} - 1$$

Example: Solve  $2\cos^2 x + 3\sin x = 0$

Solution: The equation

$$2\cos^2 x + 3\sin x = 0$$

$$\Rightarrow 2(1 - \sin^2 x) + 3\sin x = 0$$

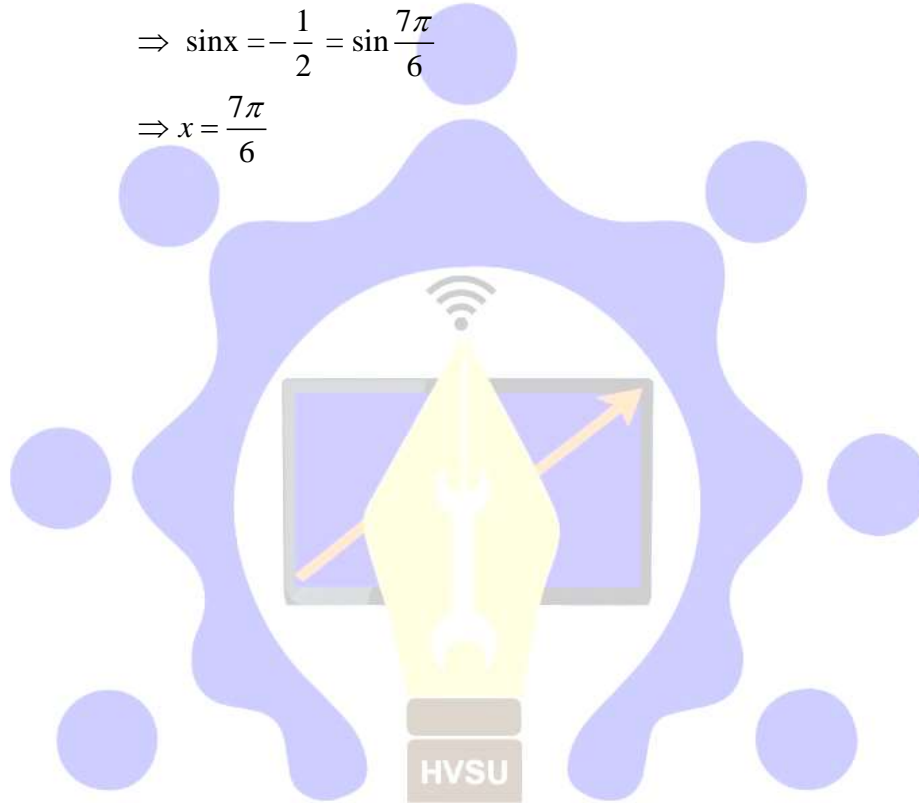
$$\Rightarrow (2\sin x + 1)(\sin x - 2) = 0$$

$$\Rightarrow \sin x = -\frac{1}{2} \text{ or } \sin x = 2$$

$$\Rightarrow \sin x = 2 \text{ is not possible}$$

$$\Rightarrow \sin x = -\frac{1}{2} = \sin \frac{7\pi}{6}$$

$$\Rightarrow x = \frac{7\pi}{6}$$



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